Solutions

Exam 3
Chapters 5 and 6

Answer the following questions. *You must show your work to receive full credit.* Be sure to make reasonable simplifications. Indicate your final answer with a $\boxed{\text{box}}$.

1. (5 points) Explain in words what the definite integral of a function represents and how we approximate it.

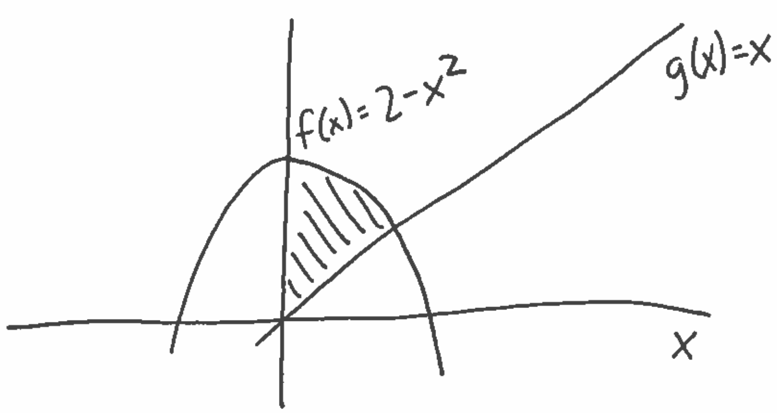
The definite integral $\int_a^b f(x)\,dx$ represents the total change in an antiderivative of f ~~from~~ on the interval $[a,b]$. We approximate it with Riemann Sums.

2 (5 points) A spaceship is traveling through space at a rate of $f(t) = t^2 + 17$ light years per minute for $0 \le t \le 4$ where t is measured in minutes. Use a right Riemann sum with $n = 2$ subintervals to approximate the the total distance that the ship covers. Make sure to give units.

$n=2$, so $\Delta t = \frac{4-0}{2} = 2$.

$f(2)\cdot 2 + f(4)\cdot 2 = 42 + 66 = 108$ light years

3. (5 points) Consider the graph below. Represent the indicated area as a definite integral.



$2-x^2 = x$

$x^2+x-2=0$

$(x+2)(x-1)=0$

$x=-2, 1$

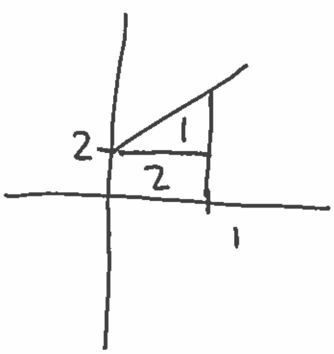
$\int_0^1 2-x^2-x\,dx.$

4. (10 points) Water is leaking from your city pool at a rate of $g(t) = \frac{5}{t} - \frac{3}{t^2}$ gallons per minute, where t is in minutes. How much water leaks from the pool in the second hour?

$$\int_{60}^{120} \frac{5}{t} - \frac{3}{t^2}\,dt = 5\ln t + \frac{3}{t}\Big|_{60}^{120}$$

$$= 5\ln(120) + \frac{1}{40} - \left(5\ln(60) + \frac{1}{20}\right).$$

5. (5 points) Use the graph of the function $f(x) = 2x + 2$ to evaluate $\int_0^1 f(x)dx$.



$$\int_0^1 f(x)dx = 3$$

6. (6 points) Find the antiderivative $F(x)$ of the function $f(x) = 3x^2 + e^x$ which satisfies $F(0) = 2$.

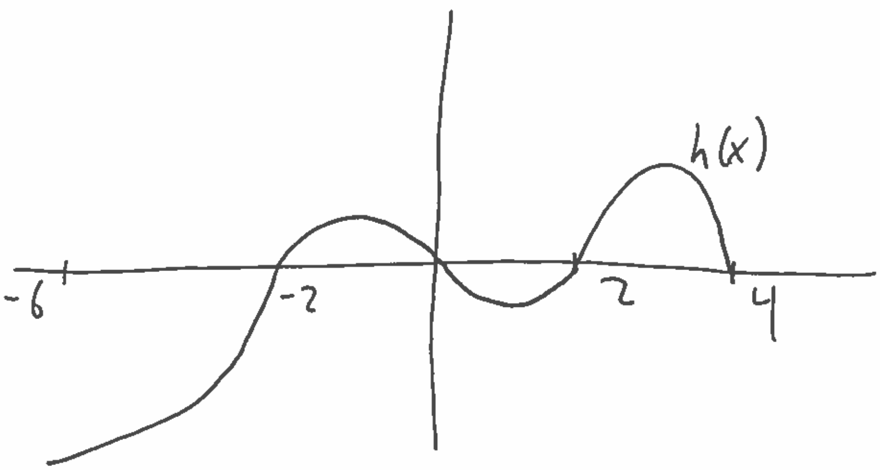
$$\int 3x^2 + e^x dx = x^3 + e^x + C$$

$2 = F(0) = 0^3 + e^0 + C = 1 + C$. Thus $C = 1$.

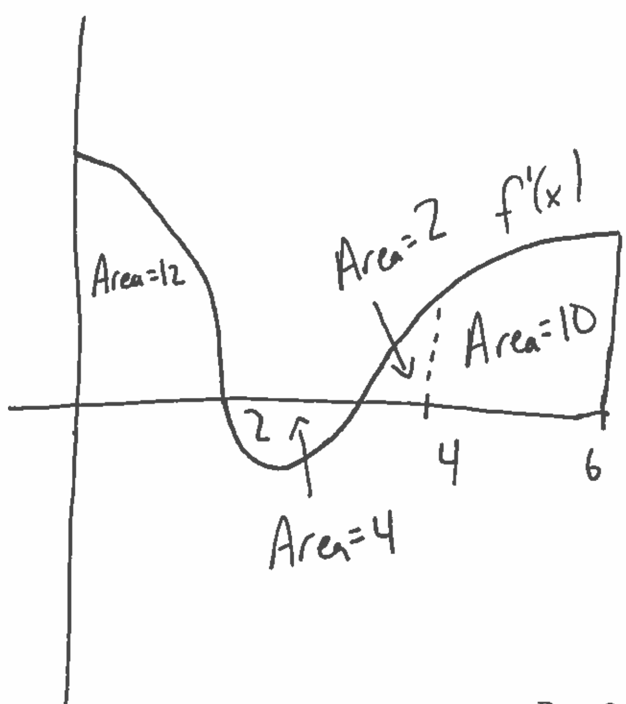
So $F(x) = x^3 + e^x + 1$.

7. (3 points each) Consider the graph of $h(x)$ below. Determine if each of the following is positive, negative or zero.

(a) $\int_{-6}^{0} h(x)dx < 0$ (b) $\int_{-2}^{2} h(x)dx = 0$ (c) $\int_{-6}^{4} h(x)dx < 0$



8. (10 points) The derivative $f'(x)$ is graphed below. Fill in the table of values for $f(x)$ given that $f(0) = 10$.



x	0	2	4	6
f(x)	10	22	20	30.

$f(2) = f(0) + \int_0^2 f'(x)dx = 22$

$f(4) = f(2) + \int_2^4 f'(x)dx = 20$

$f(6) = f(4) + \int_4^6 f'(x)dx = 30$

9. (4 points) Find the derivative of the function $g(x) = \ln(t^3 + 1)$. Make sure you show work and mention which rule your are using to solve this. (Hint: See next problem)

Chain Rule $\quad g'(x) = \frac{1}{t^3+1} \cdot 3t^2$

10. (6 points) Evaluate $\int_0^{10} \frac{3t^2}{t^3+1} dt$.

$$\int_0^{10} \frac{3t^2}{t^3+1} dt = \ln(t^3+1)\Big|_0^{10} = \ln(1001) - \ln(1) = \ln(1001).$$

11. (5 points) Find the indefinite integral $\int (3x^9 + e^{2x} - \frac{3}{x})dx$.

$$\frac{3}{10}x^{10} + \frac{1}{2}e^{2x} - 3\ln|x| + C$$

Bonus Question. Draw a picture.